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GIMRADA Research Note No. 10

THE INFLUENCE OF ATMOSPHERIC REFRACTION ON  
DIRECTIONS MEASURED TO AND FROM A SATELLITE

By Hellmut H. Schmid

7 February 1963

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FORT BELVOIR VA

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FORT BELVOIR, VIRGINIA

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ON DIRECTIONS MEASURED TO AND FROM A SATELLITE

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## SUMMARY

After a discussion of the basic shortcoming of the mathematical expression generally used to compute atmospheric refraction in connection with satellite observations, an approach is presented for computing corrections for refraction for both the geodetic satellite triangulation and the topographic photogrammetric satellite method.

For target points outside the effective atmosphere, the refraction is obtained as astronomical refraction minus a correction angle. This correction angle is a function of the corresponding astronomical refraction and at the same time, constitutes the amount of refraction encountered if a photogrammetric camera is placed outside the effective atmosphere.

This correction angle is insensitive to changes in astronomical refraction. It is, therefore, concluded that the determination of refraction from a ground station to a satellite is mainly affected by the error made in determining astronomical refraction, and directions observed at a satellite are affected only insignificantly by refraction anomalies. In addition, approximation formulas are given for use in connection with moderately sized zenith angles.

THE INFLUENCE OF ATMOSPHERIC REFRACTION  
ON DIRECTIONS MEASURED TO AND FROM A SATELLITE

I. INTRODUCTION

The geodetic-photogrammetric community is considering a twofold purpose for satellites. For supporting classic geodetic triangulation schemes, the satellite is used as an auxiliary target point and the direction to the satellite is measured from the ground. For topographic mapping, the satellite serves as the carrier for a precision photogrammetric camera system--measuring the direction from the satellite to the ground. For both objectives, the accuracy requirements are such that the problem of atmospheric refraction must be considered.

II. GENERAL PRINCIPLES OF THE REFRACTION GEOMETRY

Solutions for the numerical evaluation of the problem have been published in the past [Brown, 1957; Case, 1962; Holland, 1961; Jones, 1961; and Schmid, 1959]. These solutions and this one are based on the classic refraction geometry as shown on page 2. A representative expression for these solutions is:

$$\tan z_a = \frac{\sin C_s}{\cos C_s - \frac{r_a}{r_s}} \quad (1)$$

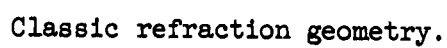
where

$$C_s = (z)_a + r_\infty - \Delta r \text{ and } \Delta r = \sin^{-1} \frac{k}{r_s}$$

$$k = n_a r_a \sin(z)_a \quad r_\infty \text{ denotes the astronomical refraction}$$

Correctly, the practical value of an expression of the form of formula (1) has been questioned. The strength of such a solution has been doubted because of the usually small central angle  $C_s$  (e. g. [Case, 1962]). More correctly, the weakness of these solutions is caused by the ratio:  $\frac{r_s}{r_s - r_a}$

A differentiation of formula (1) with respect to  $C_s$  gives:



$$\frac{1}{\cos^2 z_a} \Delta z_a = \frac{\frac{r_s}{r_a \cos C_s} - 1}{\frac{r_s}{r_a} + \frac{r_a}{r_s \cos C_s} - 2} \Delta C_s \quad (2)$$

if  $C_s$  is a small angle  $\cos C_s \approx 1$ . Thus, one obtains:

$$\frac{1}{\cos^2 z_a} \Delta z_a = \frac{\frac{r_s}{r_a} - 1}{\frac{r_s}{r_a} + \frac{r_a}{r_s} - 2} \Delta C_s \quad (3)$$

or

$$\Delta z_a = \frac{r_s}{r_s - r_a} \cos^2 z_a \Delta C_s \quad (4)$$

or

$$\Delta z_a = \left(1 + \frac{r_a}{H_s}\right) \cos^2 z_a \Delta C_s \quad (5)$$

where  $H_s$  is the height of the orbiting satellite.

It follows from formula (5) that formula (1) becomes the more useful the greater the height of the orbiting satellite. Corresponding error coefficients are given in Table I.

Table I. Numerical Evaluation of Formula (5)  
for  $\Delta z_a$  for Various Heights of Satellite

Zenith Angle	$H_s = 100 \text{ km}$	$H_s = 300 \text{ km}$	$H_s = 1,000 \text{ km}$	$H_s = r_a$	$H_s = 10 r_a$
(o)	(sec/sec)	(sec/sec)	(sec/sec)	(sec/sec)	(sec/sec)
0	64.70	21.00	7.00	2.00	1.10
15	60.37	19.59	6.53	1.87	1.03
30	48.52	15.75	5.25	1.50	0.82
45	32.35	10.50	3.50	1.00	0.55
60	16.18	5.25	1.75	0.50	0.28
75	4.33	1.41	0.47	0.13	0.07

However, both the unfavorable error propagation and the necessity of computing with the small central angle  $C_s$  can be avoided without losing the strength of the geometry inherent in a spherical earth solution.

From the classic refraction geometry (page 2):

$$z_a = (z)_a + \dot{r}_a = (z)_a + r_\infty - \sigma \quad (6)$$

and 
$$z_s = (z)_s + \dot{r}_s = (z)_s + \sigma \quad (7)$$

From formulas (6) and (7), it follows directly that:

$$\dot{r}_a + \dot{r}_s = r_\infty \quad (8)$$

#### A. Directions Measured from Ground to Satellite (Spherical Earth).

Again directly from the classic refraction geometry:

$$k = nr \sin(z) = n_a r_a \sin(z)_a \quad (9)$$

$$\Delta k = r_a \sin C_\infty = r_a \sin [(z)_a + r_\infty] \quad (10)$$

and 
$$k^* = r_a \{ n_a \sin(z)_a - \sin [(z)_a + r_\infty] \} \quad (11)$$

$$\sigma = \sin^{-1} \frac{k^*}{d} = \frac{k^*}{d} \rho'' \quad (12)$$

where d is the distance between ground and satellite. (All linear parameters are in meters.)

A classic expression [Jordan, 1939] for the relation between astronomical refraction ( $r_\infty$ ) and index of refraction (n) is:

$$r_\infty = (n-1)(1-s) \tan(z) - \left[ s(n-1) - \frac{(n-1)^2}{2} \right] \tan^3(z) \quad (13)$$

where 
$$s = \frac{P}{g r \textcircled{H}} = 0.001255 \quad (14)$$

P = weight of mercury column  
0.76 m length where the  
density of mercury = 13.596

$\textcircled{H}$  = density of air = 0.001293

r = 6,370,000 m

which may be written as:

$$s = \frac{RT_0}{r} = 0.001255 \quad (15)$$

where  $R$  = gas constant = 29.2745

$$T_0 = 273.16 \text{ K}^0$$

Formula (13) can be rearranged to read:

$$r_\infty = (n_a - 1) \tan(z)_a \left[ 1 - s - \tan^2(z)_a \left( s - \frac{n_a - 1}{2} \right) \right] \quad (16)$$

A first approximation for  $(n_a - 1)$  is obtained with

$$n_a - 1 = \frac{r_\infty}{\tan(z)_a} \quad (17)$$

Substituting formula (17) into the parenthetic terms of formula (16), one obtains:

$$n_a = 1 + \frac{r_\infty}{I \tan(z)_a} \quad (18)$$

where

$$I = 1 - \left( \frac{s}{\cos^2(z)_a} - \frac{r_\infty \tan(z)_a}{2} \right) \quad (19)$$

Substituting formula (18) into formula (11)

$$k^* = r_a \left[ \sin(z)_a (1 - \cos r_\infty) + \cos(z)_a \left( \frac{r_\infty}{I} - \sin r_\infty \right) \right] \quad (20)$$

Because  $r_\infty$  is a small angle,  $\sin r_\infty \approx r_\infty$ . Therefore, formula (20) can be written as:

$$k^* = r_a \left[ \sin(z)_a (1 - \cos r_\infty) + r_\infty \cos(z)_a \left( \frac{1}{I} - 1 \right) \right] \quad (21)$$

Because the parenthetic term in formula (19) is small, one can write:

$$I = 1 - x \text{ and consequently } \left( \frac{1}{I} - 1 \right) = 1 + x - 1 = x$$

where

$$x = \frac{s}{\cos^2(z)_a} - \frac{r_\infty \tan(z)_a}{2} \quad (22)$$

With the corresponding substitution, formula (21) reduces to:

$$k^* = r_a \left[ (\sin(z)_a (1 - \cos r_\infty) + r_\infty \left( \frac{s}{\cos(z)_a} - \frac{r_\infty \sin(z)_a}{2} \right) \right] \quad (23)$$

which can be arranged into the form:

$$k^* = r_a \sin(z)_a \left[ (1 - \cos r_\infty) - \frac{r_\infty^2}{2} \right] + \frac{r_a r_\infty s}{\cos(z)_a} \quad (24)$$

$r_\infty$  being small,  $\cos r_\infty \approx 1 - \frac{r_\infty^2}{2}$  and, therefore,

$$\left[ (1 - \cos r_\infty) - \frac{r_\infty^2}{2} \right] = 0 \quad (25)$$

Thus, one obtains finally:

$$k^* = \frac{r_a r_\infty s}{\cos(z)_a} \quad (26)$$

and with formula (15)

$$k^* = RT_0 \left( 1 + \frac{H_a}{r} \right) \frac{r_\infty}{\cos(z)_a} \approx RT_0 \frac{r_\infty}{\cos(z)_a} \quad (H_a \text{ being } < 8,000 \text{ m}) \quad (27)$$

With formula (12), the satellite refraction  $\sigma$  is obtained as

$$\sigma = \frac{r_a r_\infty'' \cdot s}{d \cos(z)_a} = \frac{r_a \cdot s \cdot r_\infty''}{H_s \left[ 1 - \frac{H_s \tan^2(z)_a}{12.5 \cdot 10^6} \right] - H_a} \quad (28)$$

Because of the insensitivity of this expression with respect to  $d$ , for moderately sized zenith angles it is possible to set

$$d \approx \frac{H_s - H_a}{\cos(z)_a} \quad (\text{flat earth approximation}) \quad (29)$$

Consequently:\*

$$\sigma \approx \frac{r_a s}{H_s - H_a} r_\infty'' \approx RT_0 \frac{r_\infty''}{(H_s - H_a)} \quad (30)$$

---

\* For limitations of formula (30), compare results in Table IV.

In order to study the propagation of errors of  $r_a$ ,  $r_\infty$ , and  $(z)_a$  with respect to  $k^*$ , formula (26) is differentiated:

$$\frac{\partial k^*}{\partial r_a} = \frac{r_\infty s}{\cos(z)_a} \partial r_a = \frac{k^*}{r_a} \partial r_a \quad (31)$$

$$\frac{\partial k^*}{\partial (z)_a} = \frac{r_a r_\infty s \tan(z)_a}{\cos(z)_a} \frac{\partial (z)_a''}{\rho''} = k^* \tan(z)_a \frac{\partial (z)_a''}{\rho''} \quad (32)$$

$$\frac{\partial k^*}{\partial r_\infty} = \frac{r_a s}{\cos(z)_a} \frac{\partial r_\infty''}{\rho''} = \frac{k^*}{r_\infty} \frac{\partial r_\infty''}{\rho''} \quad (33)$$

and accordingly with formulas (28) and (30)

$$\frac{\partial \sigma}{\partial r_a} = \frac{\sigma}{r_a} \partial r_a \quad \text{and} \quad \frac{\partial \sigma}{\partial (H_S - H_a)} = - \frac{\sigma}{H_S - H_a} \partial (H_S - H_a) \quad (34)$$

$$\frac{\partial \sigma}{\partial (z)_a} = \sigma \tan(z)_a \frac{\partial (z)_a}{\rho''} \quad (35)$$

$$\frac{\partial \sigma}{\partial r_\infty} = \frac{\sigma}{r_\infty} \partial r_\infty'' \quad (36)$$

Mean astronomical refraction can be computed according to de Ball [1906].

$$r_{\infty m} = A'_0 \tan(z)_a - A'_1 \tan^3(z)_a + A'_2 \tan^5(z)_a - A'_3 \tan^7(z)_a + \dots \quad (37)$$

A similar formula, useful for extremely large zenith distances, was developed by Garfinkel [1944].

de Ball's formula was used in Table II which gives in column (5) mean astronomical refraction  $r_{\infty m}$  for various zenith distances.

Astronomical refraction  $r_\infty$  is then obtained as:

$$r_\infty = r_{\infty m} \cdot W \quad (38)$$

where  $W$  is a meteorological correction factor. For comparison, column (6) in Table II gives astronomical refraction for  $t = 10^\circ \text{ C}$ .

$$W = \frac{P_a}{P_o} \frac{(1 + \beta t_o)}{1 + \beta t_a}$$

$$P_o = 760 \text{ mm/Hg}$$

$$t_0 = 0^\circ \text{ C}$$

$$\beta = 0.003665$$

Table II. Mean Astronomical Refraction

(1) Zenith Angle (z) <sub>a</sub>	(2) A <sub>0</sub> ' tan(z) <sub>a</sub> "	(3) -A <sub>1</sub> ' tan <sup>3</sup> (z) <sub>a</sub> "	(4) A <sub>2</sub> ' tan <sup>5</sup> (z) <sub>a</sub> "	(5) r <sub>∞m</sub> "	(6) r <sub>∞</sub> for t <sub>a</sub> = 10° C
0	0	0	0	0	0
15	16.104	-0.001	0	16.10	15.53
30	34.669	-0.013	0	34.69	33.46
45	60.101	-0.067	0	60.03	57.90
60	104.097	-0.346	0.003	103.75	100.08
75	224.298	-3.461	0.153	220.99	213.27

Table III. Numerical Evaluation of  
Formulas (20), (26), (31), (32), (33), and (41)  
Computed with r<sub>∞</sub> = r<sub>∞m</sub> and H<sub>a</sub> = 0

(1) Zenith Angle (z) <sub>a</sub>	(2) Formula (20)	(3) k*(m) Formula (26)	(4) Formula (40) or (41)	(5) $\frac{\partial k^*}{\partial r_a}$ m/m	(6) $\frac{\partial k^*}{\partial (z)_a}$ m/sec	(7) $\frac{\partial k^*}{\partial r_\infty}$ m/sec
0	0	0	0	0	0	0
15	0.67	0.65	0.65	0.0000001	0.000001	0.040
30	1.55	1.55	1.55	0.0000002	0.000004	0.045
45	3.26	3.29	3.29	0.0000005	0.000016	0.055
60	8.07	8.04	8.06	0.0000013	0.000068	0.077
75	33.54	33.07	33.56	0.0000052	0.000598	0.158

From column (7) of Table III, it is seen that the computation of k\* is insensitive to an error in r<sub>∞</sub>. Table II shows that the first-order term of formula (37) gives a good approximation for r<sub>∞m</sub>. Consequently, formula (26) can be written as:

$$k^* = r_a s A_0' \frac{\tan(z)_a}{\cos(z)_a} \cdot W \quad (39)$$

or with formula (27) as:

$$k^* = RT_0 A'_0 \frac{\tan(z)_a}{\cos(z)_a} \cdot W \quad (40)$$

with

$$r = 6,370,000$$

$$RT_0 = 7996.622420$$

$$A'_0 = 0.00029137566$$

$$k^* = 2.330 \frac{\tan(z)_a}{\cos(z)_a} \cdot W \quad (41)$$

The adequacy of formula (41) can be judged from Table III by comparing columns (2) and (4).

Again with formula (12), one obtains from formula (41):

$$\sigma'' = \frac{2.3300 \tan(z)_a}{d \cos(z)_a} \cdot \rho'' \cdot W \quad (42)$$

and in accordance with formula (29):

$$\sigma'' = \frac{2.3300}{H_s - H_a} \tan(z)_a \cdot \rho'' \cdot W \quad (43)$$

For limitations of formula (43), compare corresponding results in Table IV.

Table IV shows the computation of  $\sigma$  for various zenith angles and heights of the orbiting satellite with formulas (28), (30), (42), and (43) for  $H_a = 0$ . Table V shows the corresponding error coefficients for formulas (34), (35), and (36) using  $\sigma$  computed with formula (42).

Table IV. Numerical Values for  $\sigma$  for Various Zenith Angles and Heights of Satellite Computed with Formulas (28), (30), (42), and (43)

Zenith Angle o	$\sigma''$									
	$H_s = 100 \text{ km}$			$H_s = 300 \text{ km}$			$H_s = 1000 \text{ km}$			
	Computed with Formulas (28) (30) (42) (43)			Computed with Formulas (28) (30) (42) (43)			Computed with Formulas (28) (30) (42) (43)			
0	0	0	0	0	0	0	C	0	0	0
15	1.29	1.29	1.29	0.43	0.43	0.43	C.13	0.13	0.13	0.13
30	2.78	2.77	2.78	0.93	0.92	0.93	C.28	0.28	0.28	0.28
45	4.84	4.80	4.84	1.64	1.60	1.64	C.51	0.48	0.51	0.48
60	8.50	8.29	8.51	2.95	2.76	2.95	C.98	0.83	0.98	0.83
75	19.67	17.67	19.70	7.47	5.89	7.48	2.87	1.77	2.88	1.79

Zenith Angle o	$\sigma''$									
	$H_s = r_a$			$H_s = 10 \text{ r}_a$						
	Computed with Formulas (28) (30) (42) (43)			Computed with Formulas (28) (30) (42) (43)						
0	0	0	0	0	0	0	0	0	0	0
15	0.02	0.02	0.02	0.002	0.002	0.002	0.002	0.002	0.002	0.002
30	0.05	0.04	0.05	0.005	0.004	0.005	0.005	0.005	0.004	0.004
45	0.09	0.08	0.09	0.010	0.008	0.010	0.010	0.010	0.008	0.008
60	0.20	0.13	0.20	0.025	0.013	0.025	0.025	0.013	0.025	0.013
75	0.73	0.28	0.73	0.102	0.028	0.102	0.102	0.028	0.102	0.028

Table V. Error Coefficients

(1) Zenith Angle o	(2) $\frac{\partial \sigma}{\partial r_a}$ (sec/m) $H_a = 100 \text{ km}$	(3) $\frac{\partial \sigma}{\partial H_s}$ (sec/m)	(4) $H_s = 100 \text{ km}$	(5) $H_s = 300 \text{ km}$	(6) $\frac{\partial \sigma}{\partial r_s}$ (sec/sec) $H_s = 1000 \text{ km}$	(7) $H_s = r_a$	(8) $H_s = 10r_a$
0	0	0	0	0	0	0	0
15	0.0000002	0.0000129	0.080	0.027	0.0081	0.0012	0.00012
30	0.0000004	0.0000278	0.080	0.027	0.0081	0.0014	0.00014
45	0.0000008	0.0000481	0.081	0.027	0.0085	0.0015	0.00017
60	0.0000013	0.0000851	0.082	0.028	0.0094	0.0019	0.00024
75	0.0000031	0.0001970	0.089	0.034	0.0130	0.0033	0.00046

B. Directions Measured from Satellite to Ground (Spherical Earth).

From the illustration (page 2):

$$(z)_a = 180 - [(z)_s - C_s + r_a] \quad (44)$$

$$(r_a + H_r) = \frac{r_s \sin (z)_s}{\sin [(z)_s - C_s]} \quad (45)$$

where the refractive height

$$H_r = \frac{k^*}{\sin [(z)_s - C_s]} \quad (46)$$

Consequently,

$$r_a = \frac{r_s \sin (z)_s - k^*}{\sin [(z)_s - C_s]} \quad (47)$$

Because of the insensitivity of the computation of  $k^*$  to a change in  $r_a$ , it is adequate to write:

$$r_a = \frac{r_s \sin (z)_s}{\sin [(z)_s - C_s]} \quad (48)$$

Substituting formula (44), neglecting  $r_\infty$ , and substituting formula (48) into formula (26):

$$k^* = \frac{-2r_s r_\infty \cdot s \sin(z)_s}{\sin 2 [(z)_s - C_s]} \quad (49)$$

The differentiation of formula (49) gives:

$$\frac{\partial k^*}{\partial r_s} = \frac{k^*}{r_s} \partial r \quad (50)$$

$$\frac{\partial k^*}{\partial (z)_s} = \frac{k^*}{\rho''} \{ \operatorname{ctn} (z)_s - 2 \operatorname{ctn} 2 [(z)_s - C_s] \} \frac{\partial (z)_s''}{\rho''} \quad (51)$$

$$\frac{\partial k^*}{\partial C_s} = \frac{-k^*}{\rho''} 2 \operatorname{ctn} 2 [(z)_s - C_s] \frac{\partial C_s''}{\rho''} \quad (52)$$

$$\frac{\partial k^*}{\partial r_\infty} = \frac{k^*}{r_\infty} \frac{\partial r_\infty''}{\rho''} \quad (53)$$

In practice, instead of using formula (49), it will be from the computational standpoint, advantageous to compute with formula (44) neglecting  $r_\infty$ .

$$(z)_a = 180 - [(z)_s - C_s] \quad (54)$$

$k^*$  is then computed with formula (26) as:

$$k^* = \frac{r_a r_\infty s}{\cos [(z)_s - C_s]} \quad (55)$$

or with formula (41) as:

$$k^* = 2.3300 \frac{\tan [(z)_s - C_s]}{\cos [(z)_s - C_s]} W \quad (56)$$

Using formula (56), one obtains with formula (42)

$$\sigma'' = \frac{2.3300 \tan [(z)_s - C_s]}{d \cos [(z)_s - C_s]} \rho'' \cdot W \quad (57)$$

or in analogy to formula (43)

$$\sigma'' = \frac{2.3300}{H_S - H_A} \tan [(z)_S - C_S] \rho'' \cdot W \quad (58)$$

For limitations of formula (58), compare the corresponding result in Table VI.

Formulas (49), (56), (57), and (58) are numerically evaluated in Table VI. The result is, as it obviously has to be, in agreement with the corresponding results presented in Tables III and IV.

Table VI. Numerical Evaluation of  
Formulas (49), (56), (57), and (58)

(1) Zenith (z) <sub>a</sub>	(2) Angle [(z) <sub>s</sub> -C <sub>s</sub> ] °	(3) k* (m) Formula (49)	(4) Formula (56)	(5) σ'' Formula (57) H <sub>s</sub> = 100 km	(6) Formula (58)	(7) H <sub>r</sub> (m) Formula (46)
0	180	0	0	0	0	0
15	165	0.65	0.65	1.29	1.29	2.49
30	150	1.55	1.55	2.78	2.77	3.10
45	135	3.29	3.29	4.84	4.81	4.65
60	120	8.03	8.06	8.52	8.33	9.27
75	105	33.08	33.59	19.71	17.94	34.24

### C. Formulas Necessary for Computing Directions to and from a Satellite.

Corrected for refraction, the formulas necessary for computing directions to and from a satellite are:

$$z_a = (z)_a + r_{\infty} - \sigma \quad (6)$$

$$z_s = (z)_s + \sigma \quad (12)$$

$$\sigma = \frac{2.33}{d} \frac{\tan (z)_a}{\cos (z)_a} \rho'' \cdot W = \frac{2.33}{d} \frac{\tan [(z)_s - C_s]}{\cos [(z)_s - C_s]} \cdot \rho'' \cdot W \quad \begin{matrix} (42) \\ \text{and} \\ (57) \end{matrix}$$

or

$$\sigma = \frac{2.33}{H_S - H_A} \tan (z)_a \cdot \rho'' \cdot W = \frac{2.33}{H_S - H_A} \tan [(z)_s - C_s] \cdot \rho'' \cdot W \quad \begin{matrix} (43) \\ \text{and} \\ (58) \end{matrix}$$

$(z)_a$  and  $(z)_s$  are the measured directions at the ground or at the satellite, respectively. Because  $\sigma$  is a small angle, only a few digits must be carried in the corresponding computations.

$r_\infty$  can be computed with formula (37) or according to Garfinkel [1944],  $W$  is obtained from formula (38).

### III. DISCUSSION

Formula (6) shows that a direction measured on the ground to a satellite must be corrected for astronomical refraction and a certain parallactic correction  $\sigma$  which is functionally related to astronomical refraction. A direction measured at a satellite needs only to be corrected for the same parallactic correction  $\sigma$  (formula (7)).

The computation of astronomical refraction is, as Table I demonstrates, sufficiently proportional to  $\tan(z)$  (flat earth approximation) for moderately sized zenith distances, and for larger zenith distances, higher order terms must be carried in order to satisfy the accuracy requirements for geodetic satellite triangulation purposes. In practice, it will be necessary to either determine  $r_\infty$  independently by elevation angle measurements to stars or by indirectly considering the astronomical refraction by interpolating the photographed satellite image into the background of the surrounding star imagery.

The computation of  $k^*$  and, correspondingly, of the angle  $\sigma$  is, as shown in Tables III and V, extremely insensitive to a change in  $r$ , similarly insensitive to changes in  $(z)_a$  of  $[(z)_s - C_s]$  respectively, and fairly insensitive to changes in  $r_\infty$ .

Consequently, satellite photogrammetry will not encounter any difficulties in computing the corresponding refraction corrections. Because  $\sigma$  is a small angle (data in Table IV or [1959]), it is feasible to first compute the photogrammetric triangulation without any refraction correction. Such a result certainly will assure more than the necessary fidelity in the geometric relation between ground and satellite, for computing the corresponding refraction corrections in the last iteration cycle.

In order to give some idea of the error caused by neglecting refraction for satellite photogrammetry, the formula (46) for refractive height  $H_r$  (page 2) is given and corresponding values are presented in Table VI, column 7. In practice,  $[(z)_s - C_s]$  will be  $> 120^\circ$  resulting in an  $H_r < 9$  m. Assuming an error in elevation of about  $1/20,000$  of flying height as a practical limit in photogrammetric triangulation, it becomes evident that for satellites orbiting

at 200 to 300 km height, it is hardly necessary to consider refraction at all.

Concluding, it should be mentioned that the results obtained by Jones [1961] have given rise to wrong conclusions. His error coefficients have no physical significance, but are essentially the values obtained from formula (5) in Table I.

Holland [1961] is considering the "Effects of Atmospheric Refraction on Angles Measured from a Satellite." Qualitatively, he obtains a correct conclusion, but his approach lacks geometrical strength and, therefore, his formulas and numerical results are not adequate for computing either satellite refraction or corresponding error propagation coefficients.

As seen from formula (11),  $k^*$  is the difference between two large numbers. It is, therefore, necessary to assure geometrical fidelity when the relation between the index of refraction and the corresponding astronomical refraction is introduced. Therefore, for this phase of the problem the use of flat earth expressions is not adequate. Holland obtains values for the satellite refraction and for the corresponding error propagation coefficients which are approximately one to two magnitudes too small.

Case [1962] correctly points out that the basic formula (1) is not suited for numerical work. However, he attributes the difficulties to the small center angle  $C_s$  and, consequently, supports Jones' misleading conclusion concerning the propagation of an error in astronomical refraction. By using Holland's extended formula for astronomical refraction, Case obtains realistic values for  $\sigma$ . A comparison of his results in Table I, column (5), with Table IV of this paper shows that Case's values are slightly too large for large zenith angles, as it must be, because Holland's extended formula for astronomical refraction neglects the term  $\frac{(n-1)^2}{2} \tan^3(z)$ . Compare formula (16) of this paper.

However, the error coefficients in the last two columns of Table 1 in [Case (1962)] are insignificant, as are the similar values in [Jones (1961)]. The results given in Table 2 in [Case (1962)], labelled "Flat-Earth Satellite Refraction," lead to correct values for satellite refraction, within the limits imposed by the approximation  $d = \frac{H_s}{\cos(z)_a}$ , namely, flat earth geometry and assuming  $H_a = 0$ . The effect of a flat earth geometry can be judged from Table VI, columns (5) and (6) in this report.

The results obtained in this paper indicate that proper considerations of refraction corrections for both the geodetic satellite

triangulation and the topographic photogrammetric satellite method are possible without any computational difficulties. Furthermore, it seems justified as long as the camera or the target point is outside of the effective atmosphere, to state that refraction cannot be made the scapegoat if in the future the need should arise to explain discrepancies in the results of photogrammetric space triangulation methods.

This statement should not detract from the problem of scintillation, which affects particularly short-duration electronic flash photography. The geometrical significance of directions measured to such targets is limited, independent of the precision of the measuring method, by the scintillation effect to an accuracy (one sigma level) of  $\pm 2$  to  $\pm 3$  seconds of arc.

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